Introduction to Indices

A power, or an index, is used to write a product of numbers very compactly. The plural of index is indices.

The following states a number of rules, or laws, which can be used to simplify expressions involving indices.

1. Powers or indices

The expression $3 \times 3 \times 3 \times 3$ can be written as 3^4.

We read this as '**three to the power four**'.

Similarly $z \times z \times z = z^3$ which is read as '**z to the power three**' or '**z cubed**'.

In the expression b^c, the index is c and the number b is called the base.

Your calculator will probably have a button to evaluate powers of numbers. It may be marked x^y.

You can use your calculator to verify that $7^4 = 2401$ and $25^5 = 9765625$

2. The laws of indices

To manipulate expressions involving indices we use rules known as the laws of indices. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own!

The following are the important laws of indices :

First law: $a^m \times a^n = a^{m+n}$ [Known as **Product Law**]

When expressions with the same base are multiplied, the indices are added.

Example:

We can write $7^6 \times 7^4 = 7^{6+4} = 7^{10}$

Second Law :$a^m \div a^n = a^{m-n}$ [Known as **Quotient Law**]

When expressions with the same base are divided, the indices are subtracted.

Example:

We can write $8^5 \div 8^3 = 8^{5-3} = 8^2$

Third law: $(a^m)^n = a^{m\,n}$

Note that m and n have been multiplied to yield the new index m n.

Example:

$(6^4)^2 = 6^{4\times2} = 6^8$

Fourth law: $(xy)^m = x^m.y^m$

Example:

$(2\cdot3)^2 = 2^2 X 3^2 = 36$

Fifth law: $(\frac{x}{y})^m = \frac{x^m}{y^m}$ [y≠0]

Example:

$(\frac{3}{2})^7 = \frac{3^7}{2^7}$

Sixth law: $x^0 = 1$ **[** provided x≠0]

[Note :- 0^0 is not defined.]

Seventh law: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Eighth Law: If $a^x = a^y$ then **x = y,** provided **a ≠ 0,±1**

If $M = a^n$ ⇨ $a = M^{\frac{1}{n}}$, also if **a>0** and **a ≠** 1 ⇨ **M > 0**

i.e. M can be expressed in terms of **a** and **n,** again **a** can be expressed in terms of **M** and **n.**

Can we express **n** in terms of **M** and **a**?

We'll try to find the answer in near future.