Grade 8

Chapter 10: Factorization

In the arithmetic expression, 42=2 x 3 x 7, 2, 3 and 7 are the factors of 42. The process of writing 42 as a product of 2, 3 and 7 is called Factorization.2, 3 and 7 cannot be factorised further.

Similarly, an algebraic expression can also be written as a product of two or more factors.

For example:

$7xyz = 7. x. y. z$

$7x-21=7(x-3)$

$x^2-x=x(x-1)$

$x^2+5x-6=(x-1)(x+6)$

In the above examples, each expression is written as a product of two or more expressions and the terms in the bracket have no common factor(other than 1).

Factorization

The process of writing a given expression as a product of two or more factors is called Factorization.

In other words, Factorization is the reverse process of expansion or finding out the product.

Factorization of Algebraic Expression When a monomial is Common in Each Term:

i) Find the Greatest Common Factor (G.C.F) of the terms in the given expression.

ii) Express the given expression as the product of G.C.F. and the other factor.

The other factor (quotient) = The given expression ÷ G.C.F. (of all terms)

i.e., Expression= G.C.F x Quotient

Example

Find the Greatest Common Factor of: $15x^2$ and $3x^2$

Solution

G.C.F (H.C.F) of numerical coefficients 15 and 3 is 3.

The common variable number appearing is x and its smallest power is 2.

∴ G.C.F. of $15x^2$ and $3x^2$ is $3x^2$.

Example

Factorise: $2x^3 - 3xy$.

Solution

H.C.F of $2x^3$ and $3xy = x$

$\therefore 2x^3 - 3xy = x(2x^2 - 3y)$.

Factorization of Algebraic Expressions when a binomial is a Common Factor

When a binomial is a common factor, the algebraic expression is written as the product of binomial and the quotient obtained by dividing the given expression by the binomial factor.

Example

Factorise: $7(2a+3b)^3 - (2a+3b)^2$.

Solution

$7(2a+3b)^3 - (2a+3b)^2$

$= (2a+3b)^2 [7(2a+3b) - 1]$

$= (2a+3b)^2 [14a+21b-1]$

$= (2a+3b)(2a+3b)(14a+21b-1)$

Factorization by grouping the terms

When all the terms of the expression do not share a common factor then we break up the expression into a small group that can be factorised. Such a process is called Factorization by grouping.

Example

Factorise: $x^2+xy+5x+5y$.

Solution

As there is no common factor in all the four terms (except 1) so we group the terms such that each group has a common factor.

$$\underbrace{x^2+xy}\; + \;\underbrace{5x + 5y}$$

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We write the polynomial as x (x +y) +5(x +y)

In this form, (x+ y) is the G.C.F. Taking it common form both the groups, we get

(x+ y) (x+ 5). Hence, x^2+xy+5x+5y= (x+ y) (x+ 5).

Factorization of Binomial as the difference of two squares

In order to factorise binomial as the difference of two squares, we use the identity:

$$a^2 - b^2 = (a- b)\ (a+ b).$$

Factorization by making a perfect square

If the given algebraic expressions is in the form of $a^2 + b^2 + 2ab$ then it can be written as $(a+ b)^2$

$$a^2 + b^2 + 2ab = (a+ b)^2 = (a+ b)\ (a+ b)$$

$$a^2 + b^2 - 2ab = (a- b)^2 = (a- b)\ (a- b)$$

Factorization of quadratic trinomials by splitting the middle term

i) When the expression is of the type $x^2 + px + q$

We know that $(x+ a)(x+ b) = x^2 + (a + b)x + ab$

So, in order to factorise $x^2 + px + q$, we find two numbers a and b such that a+ b = p and a b = q.

$$x^2 + px + q = x^2 + (a + b)x + ab = (x+ a)(x+ b)$$

ii) When the expression is of the type $ax^2 + bx + c$