Grade 8

Chapter 22: Probability

What is probability?

Probability is a chance that something will happen – how likely it is that some event will happen.

So, probability of an event happening =

$$\frac{\text{Number of wanted outcomes}}{\text{Total number of outcomes}}$$

Experiment

An Operation which can produce some well defined outcomes, is called an experiment.

e.g Tossing the coin, Throwing of a die etc.

Random Experiment

An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is called a random experiment or probabilistic experiment.

e.g: Tossing a fair coin is a random experiment because if we toss a coin either a head or tail will come up. But if we toss a coin again and again then the outcome each time will not be the same.

Here head and tail are called outcome

 Tossing a Coin **(Experiment)**	  Tail Head **(Possible Outcomes)**

Event

The set of representing the desired outcomes of a random experiment is called an event. Ex : Getting head in the toss of a coin is an event.

Sample Space / Event Space

A set of all possible outcomes of an event is called a sample space. Each element of a sample space is called sample point.

e.g when a die is thrown, then sample space for this event is {1, 2, 3, 4, 5, 6}.

Types of Event

Elementary Event

An event having only one outcome of the random experiment is called an elementary event.

e.g. : In tossing of a coin, the possible outcomes, are head (H) and tail (T). Therefore,

E_1 = getting head (H) on the upper face of the coin.

E_2 = getting tail (T) on the upper face of the coin

Then E_1 and E_2 are elementary events associated with the random experiment of tossing of a coin.

- **Compound Event**

  A collection of two or more elementary events associated with an experiment is called a compound event.

  e.g : In the random experiment of tossing of two coins simultaneously, if we define the event getting exactly one head, then it is a collection of elementary events (or outcomes) HT and TH. So, it is a compound event.

- **Equally Likely Events**

  The outcomes (or events) of a random experiment are said to be equally likely, if the different outcomes have the same or equal chances of occurrence. In other words, events are said to be equally likely when we have no reason to believe that one is more likely to occur than the other.

e.g When a die is thrown, all of the faces, i.e. 1, 2, 3, 4, 5, and 6 are equally likely to appear. So, events {1}, {2},{3},{4},{5} and {6} are equally likely events.

Complement of an Event

Let E be an event in a sample space S, then complement of E is the set of all sample points of the sample space other than the sample point in E (i.e. not E) and it is denoted by E' or $\overline{E}$. = $\{n : n \in S, n \notin E\}$

Also, E and $\overline{E}$ are called complementary events.

Probability – An Experimental Approach

If E is an event that happens when an experiment is performed, then the empirical (or experimental) probability of the event is given by

$$P(E) = \frac{\text{Number of times event E occurred}}{\text{Number of times the experiment was performed}}$$

e.g If a coin is tossed 1000 times and event E_1 = head occurs 455 times and E_2 = tail occurs 545 times. Then probability of occurrence of head,

$$P(E_1) = \frac{455}{1000} = 0.455$$

And probability of occurrence of tail,

$$P(E_2) = \frac{545}{1000} = 0.545$$

Probability – A Theoretical Approach

If all the outcomes in a sample space S are equally likely and E is an event within that sample space, then the theoretical probability (or classical probability) of the event E is given by

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}$$

e.g : When a die is thrown, there are six possible outcomes.

So sample space, S = { 1, 2, 3, 4, 5, 6} and total number of outcomes is 6. Let E be the event of getting 4, then favourable outcome is only one, ie. 4. So,

Probability of getting 4, P (E) = $\frac{1}{6}$

Example

Suppose we throw a die once.

(i) What is the probability of getting a number greater than 4?
(ii) What is the probability of getting a number less than or equal to 4?

Solution

The Sample space of a die { 1, 2, 3, 4, 5, 6}.

Total number of outcomes, n(S) = 6.

(i) Let E be the event of getting a number greater than 4.
The outcomes favorable to E are 5 and 6, since 5 and 6 are greater than 4. Therefore, the number of outcomes favourable to E, n (E) = 2.

$\therefore$ P (E) = P (number greater than 4)

$= \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(ii) Let F be the event of getting a number less than or equal to 4.

Outcomes favourable to the event F are { 1, 2, 3, 4}.
So, the number of outcomes favourable to F, n (F) = 4

$\therefore$ P (E) = P (number less than or equal to 4)

$= \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$