1. Trials & Experiments

1. Trial

Performing a single activity which has known and finite numbers of outcomes is called a trial.

For example:

Rolling a die once is a trial. Its outcomes are 1, 2, 3, 4, 5, 6

Tossing a coin is a trial. Its outcomes are *Head* (*H*) and *Tail* (*T*).

Pulling a card out of a pack of 52 cards is also a trial. Its outcome is one of those 52 cards of pack.

2. Experiment

When several trials are conducted under identical conditions then it is called an experiment.

For example:

Rolling a die 10 times is an experiment.

Tossing a coin 100 times is an experiment.

3. Random Experiment

An experiment in which all possible outcomes are known and the exact output can not be predicted is called a random experiment.

2. Events

1. Elementary Event

Occurrence of an outcome under a trial is called an elementary event.

For example:

Coming of 4 when a die is rolled once is an event.

Turning up of Head when a coin is tossed is also an event.

2. Certain Event

An event which will always occur is called a certain event.

The probability of a certain event is 1 or 100%

Illustration 1:

A die is rolled once. What is the probability that one out of 1, 2, 3, 4, 5, 6 will appear?

Solution:

A die has six faces. So it is guaranteed that one of the six faces will definitely appear.

So the probability of getting 1, 2, 3, 4, 5, 6 is 1 or 100%

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3. Impossible Event

An event that will never occur is called an impossible event.

The probability of an impossible event is 0.

4. Compound Event

When an even is formed by combining two or more elementary events then it is called a compound event.

For example:

Rolling a die and tossing a coin simultaneously is a compound event.

Illustration 2:

A die is rolled once. What is the probability that a 9 will appear.

Solution:

When a die is rolled once, the possible outcomes are: 1, 2, 3, 4, 5 and 6 only. There is no face with 9.

So the probability of getting 9 is 0.

5. Complement Event

The opposite of an event is called its complement event.

For example:

If A is an event that a 3 appears when a die is rolled then $\overline{A}$ is an event that a 3 does not appear.

If p is the probability of an event then $(1 - p)$ is the probability of its complement event.

6. Sample Space

A sample space is a set of all possible outcomes of a trial.

For example:

When a die is rolled the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

7. Set of Favours (Set of Favourable Events)

A set containing the favourable outcome(s) is called the set of favours.

8. Exhaustive Events

It is the set of all outcomes of a trial. Hence sample space itself is called exhaustive events.

When exhaustive events are given, the occurrence of one of them is guaranteed.

9. Mutually Exclusive Events

If two or more events can not occur together then they are called mutually exclusive events.

If A and B are two mutually exclusive events then $p(A \cap B) = 0$.

10. Independent Events

Two events are called independent events if the outcome of one event does not influence or affect the outcome of the other.

If A and B are two independent events then $p(A \cap B) = p(A)p(B)$.

Illustration 3:

What is the sample space when a die is rolled two times?

Solution:

When a die is rolled once, the possible outcomes are: 1, 2, 3, 4, 5, 6.

So the sample space is:

S = {11, 12, 13, 14, 15, 16, 21, 22, …, 64, 65, 66}

Illustration 4:

What is the sample space when a coin is tossed three times?

Solution:

When a coin is tossed once, the possible outcomes are Head and Tail.

Hence sample space is:

S = {TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}

Illustration 5:

What is the sample space when a die and a coin are thrown simultaneously?

Solution:

When a die and a coin are thrown simultaneously, the sample space is:

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

3. Types of Probabilities

1. Classical (Mathematical) Probability

In classical probability, the probability is calculated by the ratio of numbers of elements in the set of favors to the number of elements in the sample space.

If *n(S)* is the numbers of elements in the sample space and *n(E)* is the numbers of elements in the set of favors then:

$$P(E) = \frac{n(E)}{n(S)}$$

2. Empirical (Experimental) Probability

Suppose we perform a trail and *n* is the total numbers of outcomes possible. The empirical probability of an event E happening is given by

$$P(A) = \frac{Numbers\_of\_outcome\_in\_which\_E\_can\_occur}{Total\_numbers\_of\_outcomes}$$

Thus, $P(A) = \frac{\text{Total number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{m}{n}$

And $0 \le P(A) \le 1$

If, P(A) = 0, then A is called impossible event

If, P(A) = 1, then A is called sure event

$P(A) + P(\bar{A}) = 1$

Where P(A) = probability of occurrence of A.

$P(\bar{A})$ = probability of non - occurrence of A.

Few more solved examples:

Ex.1 A box contains 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is

(i) white (ii) neither red nor white

Sol. Total number of balls in the bag = 5 + 4 + 7 = 16

∴ Total number of elementary events =16

(i) There are 7 white balls in the bag.

∴ Favorable number of elementary events = 7

Hence, P (Getting a white ball) = $\frac{\text{Total No. favourable elementary events}}{\text{Total No. of elementary events}} = \frac{7}{16}$

(ii) There are 4 balls that are neither red nor white

∴ Favorable number of elementary events = 4

Hence, P (Getting neither red not white ball) = $\frac{4}{16} = \frac{1}{4}$

Ex.2 All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting

(i) black face card (ii) a queen (iii) a black card.

Sol. After removing three face cards of spades (king, queen, jack) from a deck of 52 playing cards, there are 49 cards left in the pack. Out of these 49 cards one card can be chosen in 49 ways.

∴ Total number of elementary events = 49

(i) There are 6 black face cards out of which 3 face cards of spades are already removed. So, out of remaining 3 black face cards one black face card ban be chosen in 3 ways.

∴ Favorable number of elementary events = 3

Hence, P (Getting a black face card) = $\frac{3}{49}$

(ii) There are 3 queens in the remaining 49 cards. So, out of these three queens, on queen can be chosen in 3 ways

∴ Favorable number of elementary events = 3

Hence P (Getting a queen) = $\frac{3}{49}$

(iii) There are 23 black cards in the remaining 49 cards, So, out to these 23 black card, one black card can be chosen in 23 ways

∴ Favorable number of elementary events = 23

Hence, P (Getting a black card) = $\frac{23}{49}$

Ex.3 A die is thrown, Find the probability of

(i) prime number (ii) multiple of 2 or 3 (iii) a number greater than 3

Sol. In a single throw of die any one of six numbers 1,2,3,4,5,6 can be obtained. Therefore, the tome number of elementary events associated with the random experiment of throwing a die is 6.

(i) Let A denote the event "Getting a prime no". Clearly, event A occurs if any one of 2,3,5 comes as out come.

∴ Favorable number of elementary events = 3

Hence, P (Getting a prime no.) = $\frac{3}{6} = \frac{1}{2}$

(ii) An multiple of 2 or 3 is obtained if we obtain one of the numbers 2,3,4,6 as out comes

∴ Favorable number of elementary events = 4

Hence, P (Getting multiple of 2 or 3) = $\frac{4}{6} = \frac{2}{3}$

(iii) The event "Getting a number greater than 3" will occur, if we obtain one of number 4,5,6 as an out come.

∴ Favorable number of out comes = 3

Hence, required probability = $\frac{3}{6} = \frac{1}{2}$