Grade 8

Chapter 17 : Coordinate Geometry

HISTORICAL NOTE :

History now shows that the two Frenchmen Rene Descartes and Pierre de Fermat arrived ´ at the idea of **analytical geometry** at about the same time. Descartes' work La Geometrie was published first, in 1637, while Fermat's Introduction to Loci was not published until after his death. Today, they are considered the co-founders of this important branch of mathematics, which links algebra and geometry. The initial approaches used by these mathematicians were quite opposite. Descartes began with a line or curve and then found the equation which described it. Fermat, to a large extent, started with an equation and investigated the shape of the curve it described. This interaction between algebra and geometry shows the power of analytical geometry as a branch of mathematics. **Analytical geometry** and its use of coordinates provided the mathematical tools which enabled Isaac Newton to later develop another important branch of mathematics called **calculus**. Newton humbly stated: "If I have seen further than Descartes, it is because I have stood on the shoulders of giants."

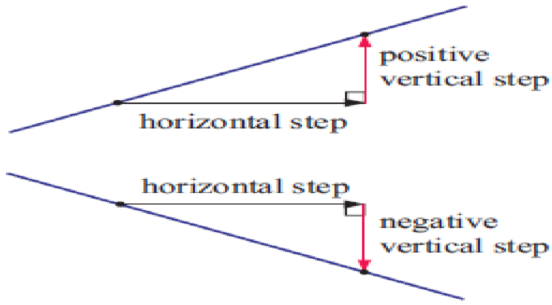

Pierre de Fermat


Réne Descartes

Gradient of a slope and a straight line :

To calculate the gradient of a line, we first choose any two distinct points on the line.
We can move from one point to the other by making a positive horizontal step followed by a vertical step.
If the line is sloping upwards, the vertical step will be positive.
If the line is sloping downwards, the vertical step will be negative.

Grade 8



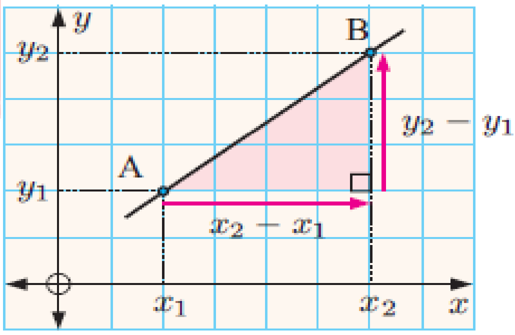
The gradient is calculated by dividing the vertical step by the horizontal step.

The gradient of a line $= \frac{\textbf{vertical step}}{\textbf{horizontal step}}$ or, $\frac{\textbf{y step}}{\textbf{x step}}$

THE GRADIENT FORMULA :

If a line passes through A(x_1, y_1) and B(x_2, y_2), then the horizontal or x-step is $x_2 - x_1$, and the vertical or y-step is $y_2 - y_1$.

The gradient of the line passing through A(x_1, y_1) and B(x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$



Note :

- The gradient of horizontal lines is **0** .
- The gradient of vertical lines is **undefined.**
- If two lines are parallel, they have **equal** gradient.
- If two lines have equal gradient, they are **parallel.**
- If the lines are **perpendicular**, their gradients are **negative reciprocals**.
- If the gradients are **negative reciprocals**, the lines are **perpendicular.**
- Three points A, B, and C are **collinear** if: gradient of AB = gradient of BC (= gradient of AC)

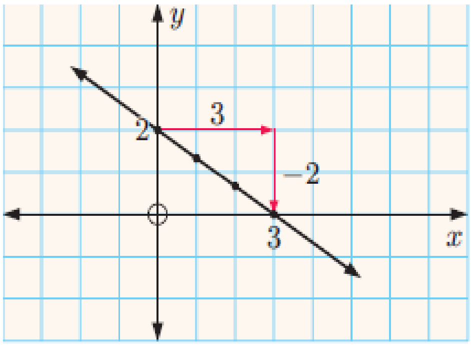
Grade 8

GRADIENT-INTERCEPT FORM :

Every straight line that is not vertical will cut the y-axis at a single point.
The y-coordinate of this point is called the y-intercept of the line.
A line with gradient m and y-intercept c has equation y = m x + c.
We call this the gradient-intercept form of the equation of a line.

For example, the line alongside has gradient = $\frac{\text{y step}}{\text{x step}} = \frac{-2}{3}$ and its y-intercept is 2.

So, its equation is $y = \frac{-2}{3}x + 2$.



GENERAL FORM :

Another way to write the equation of a line is using the general form ax + by + d = 0.
We can rearrange equations from gradient-intercept form into general form by performing operations on both sides of the equation.

For example, if $y = \frac{-2}{3}x + 2$

then 3y = -2x + 6

⇨ 2x + 3y = 6

⇨ 2x + 3y - 6=0

So, the line with gradient-intercept form $y = \frac{-2}{3}x + 2$ has general form 2x + 3y -6=0.

FINDING THE EQUATION OF A LINE

In order to find the equation, we need to know some information about the line.
Suppose we know the gradient of the line is 2 and that the line passes through (4, 1).
We suppose (x, y) is any point on the line.

The gradient between (4, 1) and (x, y) is $\frac{y-1}{x-4}$, and this gradient must equal 2.

So, $\frac{y-1}{x-4} = 2$

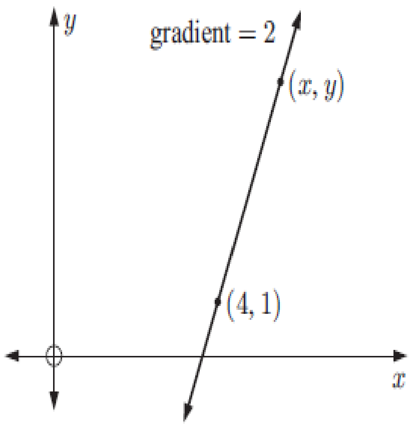
⇨ y -1 = 2(x - 4)

⇨ y -1=2x - 8

⇨ y = 2x - 7

This is the equation of the line in gradient-intercept form.

Grade 8



If a straight line has gradient m and passes through the point (x_1, y_1) then its equation is

$\frac{y-y_1}{x-x_1}$= m. We can rearrange this equation into either gradient-intercept or general form.

Midpoints and division of an interval