1. Angles

1. Definition

The rotational displacement between two lines having same initial point is called an angle between them

2. Positive (Anti Clockwise) & Negative (Clockwise) Angles

The reference line to measure an angle is positive *x*-axis. When an angle is measured in anti-clockwise direction, then its measure is a positive quantity, whereas when an angle is measured in clockwise direction its measure is a negative quantity.

3. Units of Measurements of Angles

When a point completes one cycle around a point, we say that it has covered 360 degrees or 2π radians.

1 cycle = 360^0

1 cycle = 2π radian

4. Radian to Degree Conversion

$$\theta_{\deg} = \frac{180}{\pi}\theta_{rad}$$

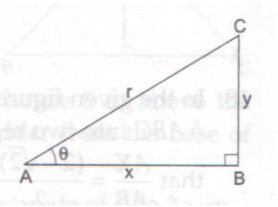
5. Degree to Radian Conversion

$$\theta_{rad} = \frac{\pi}{180}\theta_{\deg}$$

2. Basic Trigonometric Ratios

1. Trigonometric Ratios in a Right Angled Triangle

Let *ABC* be a right angled triangle right angled at *B*. Let $\angle CAB = \theta$, then side *BC* is called perpendicular (opposite side) and side *AB* is called base (adjacent side).



We define following trigonometric ratios.

- $\sin\theta = \frac{perpencdicular}{hypotenuse} = \frac{y}{r}$
- $\cos\theta = \frac{base}{hypotenuse} = \frac{x}{r}$
- $\tan\theta = \frac{perpencdicular}{base} = \frac{y}{x}$
- $\csc\theta = \frac{hypotenuse}{perpencdicular} = \frac{r}{y}$
- $\sec\theta = \frac{hypotenuse}{base} = \frac{r}{x}$
- $\cot\theta = \frac{base}{perpendicular} = \frac{x}{y}$

2. Reciprocal Relations

- $\sin\theta = \frac{1}{\csc\theta} \Rightarrow \sin\theta\csc\theta = 1$
- $\cos\theta = \frac{1}{\sec\theta} \Rightarrow \cos\theta\sec\theta = 1$
- $\tan\theta = \frac{1}{\cot\theta} \Rightarrow \tan\theta\cot\theta = 1$

3. Quotient Relation of Trigonometric Ratio

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Illustration 1:

In a triangle *ABC*, *AB* = 3cm, *BC* = 4 cm and angle *B* = 90 degree. Find $\sin A$ and $\cot C$

Solution:

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 3^2 + 4^2$$

$AC = 5$ cm

For angle A, we have:

$$\sin A = \frac{perp}{hypo} = \frac{BC}{AC}$$

$$\sin A = \frac{4}{5}$$

For angle C, we have:

$$\cot C = \frac{BC}{AB}$$

$$\cot C = \frac{4}{3}$$

Illustration 2:

In a triangle ABC, for some angle θ, $AC = x+9$ is the hypotenuse, $BC = x+8$ is the base and $AB = x+1$ is the perpendicular. Find all six trigonometric ratios for this angle.

Solution:

$$AC^2 = AB^2 + BC^2$$

$$x^2 + 18x + 81 = x^2 + 16x + 64 + x^2 + 2x + 1$$

$$\rightarrow x = 4$$

So the side are AB = 5 cm, BC = 12 cm and AC = 13 cm

$$\sin\theta = \frac{AB}{AC} = \frac{5}{13} \rightarrow \csc\theta = \frac{13}{5}$$

$$\cos\theta = \frac{BC}{AC} = \frac{12}{13} \rightarrow \sec\theta = \frac{13}{12}$$

$$\tan\theta = \frac{AB}{BC} = \frac{5}{12} \rightarrow \cot\theta = \frac{12}{13}$$

Illustration 3:

Prove that $\cos^2 A + \frac{1}{1+\cot^2 A} = 1$

Solution:

$$LHS = \cos^2 A + \frac{1}{1+\cot^2 A}$$

$$= \cos^2 A + \frac{1}{\csc^2 A}$$

$$= \cos^2 A + \sin^2 A$$

$$= 1$$

$$= RHS$$

Illustration 4:

Prove that $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$

Solution:

$$LHS = \frac{1}{1+\sin A} + \frac{1}{1-\sin A}$$

$$= \frac{1-\sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$$

$$= \frac{2}{1-\sin^2 A}$$

$$= \frac{2}{\cos^2 A}$$

$$= 2\sec^2 A$$

$$= RHS$$

The solution of equation $\cos A = \cos B$ is:

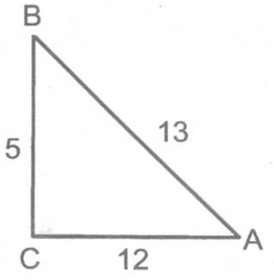
More Solved Examples

1. If cosec A = $\frac{13}{5}$ the prove than $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$.

Sol. We have cosec A $= \frac{13}{5} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

So, we draw a right triangle ABC, right angled at C such that hypotenuse AB = 13 units and perpendicular

BC = 5 units



B Pythagoras theorem,

$AB^2 = BC^2 + AC^2 \Rightarrow (13)^2 = (5)^2 + AC^2$

$AC^2 = 169 - 25 = 144$

$AC = \sqrt{144} = 12$ units

$\tan A = \frac{BC}{AC} = \frac{5}{12}$

$\sin A = \frac{BC}{AB} = \frac{5}{13}$

and $\sec A = \frac{AB}{AC} = \frac{13}{12}$

L.H.S. $\tan^2 A - \sin^2 A$

$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{25}{144} - \frac{25}{169}$

$= \frac{25(169-144)}{144 \times 169}$

$= \frac{25 \times 25}{144 \times 169}$

R.H.S. $= \sin^4 A \times \sec^2 A$

$= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2$

$= \frac{5^4 \times 13^2}{13^4 \times 12^2}$

$= \frac{5^4}{13^2 \times 12^2}$

$= \frac{25 \times 25}{144 \times 169}$

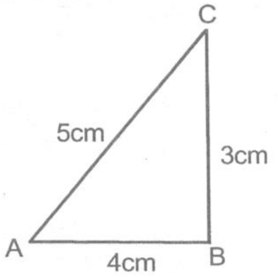
So, L.H.S. = R.H.S. **Hence Proved.**

2. In Δ ABC, right angled at B. AC + AB = 9 cm. Determine the value of cot C, cosec C, sec C.

Sol. In Δ ABC, we have

$(AC)^2 = (AB)^2 + BC^2$

$\Rightarrow$ $(9 - AB)^2 = AB^2 + (3)^2$ [$\because$ AC + AB = 9cm $\Rightarrow$ AC = 9 - AB]



$\Rightarrow$ $(81 + AB^2 - 18AB = AB^2 + 9$

$\Rightarrow$ 72 - 18 AB = 0

$\Rightarrow$ $AB = \frac{72}{18} = 4\text{cm}.$

Now, AC + AB = 9 cm

AC = 9 - 4 = 5 cm

So, $\cot C = \frac{BC}{AB} = \frac{3}{4}$, $\text{cosec } C = \frac{AC}{AB} = -\frac{5}{4}$, $\sec C = \frac{AC}{BC} = \frac{5}{3}$.

3. Prove that : $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

Sol. **L.H.S.** $\cot\theta - \tan\theta$

$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$ $\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\cos^2\theta - \sin^2}{\sin\theta\cos\theta}$ $= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta\cos\theta}$ $[\because \sin^2\theta = 1 - \cos^2\theta]$

$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta}$ $= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

Formulae for the Trigonometric Ratios of Sum and Differences of Two Angles

(1) $\sin(A+B)=\sin A\cos B+\cos A\sin B$

(2) $\sin(A-B)=\sin A\cos B-\cos A\sin B$

(3) $\cos(A+B)=\cos A\cos B-\sin A\sin B$

(4) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(5) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

(6) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

(7) $\cot(A+B)=\dfrac{\cot A\cot B-1}{\cot A+\cot B}$

(8) $\cot(A-B)=\dfrac{\cot A\cot B+1}{\cot B-\cot A}$

(9) $\sin(A+B).\sin(A-B)=\sin^2 A-\sin^2 B=\cos^2 B-\cos^2 A$

(10) $\cos(A+B).\cos(A-B)=\cos^2 A-\sin^2 B=\cos^2 B-\sin^2 A$

(11) $\tan A\pm\tan B=\dfrac{\sin A}{\cos A}\pm\dfrac{\sin B}{\cos B}=\dfrac{\sin A\cos B\pm\cos A\sin B}{\cos A\cos B}=\dfrac{\sin(A\pm B)}{\cos A.\cos B}$ $\quad\left(A\neq n\pi+\dfrac{\pi}{2}, B\neq m\pi\right)$

(12) $\cot A\pm\cot B=\dfrac{\sin(B\pm A)}{\sin A.\sin B}$ $\quad\left(A\neq n\pi, B\neq m\pi+\dfrac{\pi}{2}\right)$